Introduction: There are three half-lives that are important when considering the use of radioactive drugs for both diagnostic and therapeutic purposes. While both the physical and biological half-lives are important since they relate directly to the disappearance of radioactivity from the body by two separate pathways (radioactive decay, biological clearance), there is no half-life as important in humans as the effective half-life. As we will see shortly, this half-life takes into account not only elimination from the body but also radioactive decay. If there is ever a question about residual activity in the body, the calculation uses the effective half-life; in radiation dosimetry calculations, the only half-life that is included in the equation is the effective half-life. Let’s take a look individually at the three half-lives.
Physical half-life is defined as the period of time required to reduce the radioactivity level of a source to exactly one half its original value due solely to radioactive decay. The physical half-life is designated \( t_{\text{phys}} \) or more commonly \( t_{1/2} \). By default, the term \( t_{1/2} \) refers to the physical half-life and \( t_{\text{phys}} \) is used when either or both of the other two half-lives are included in the discussion. There are a few things to note about the \( t_{\text{phys}} \):

- The \( t_{\text{phys}} \) can be measured directly by counting a sample at 2 different points in time and then calculating what the half-life is.

- For example, if activity decreases from 100% to 25% in 24 hours, then the half-life is 12 hours since a decrease from 100% to 50% to 25% implies that 2 half-lives have elapsed.

- One can also determine graphically what the half-life is. In the diagram below, both of the lines have a half-life of 5 days, even though their activity levels are quite different.
• The range of half-lives is boundless. There are isotopes with half-lives of nsec, μsec, msec, sec, hr, min, days, weeks, months, years, centuries, millennia, and even as long as a billion years (half-life of K-40 = $1.28 \times 10^9$ years). Most of those time units would not be very useful for diagnostic or therapeutic studies and, in fact, all commercially available isotopes range from 75 sec (Rb-82) to Sr-89 (50.5 days) with all others in between those two values.

• The physical half-life is unaffected by anything that we humans can do to the isotope. High or low pressure or high or low temperature has no effect on the decay rate of a radioisotope. Perhaps taking the isotope to absolute 0 or within a degree of that temperature would affect the decay rate.
**Half Lives: Biological**

Biological Half-life is defined as the period of time required to reduce the amount of a drug in an organ or the body to exactly one half its original value due solely to biological elimination. It is typically designated $t_{biol}$ or $t_b$. There are a few things to note about the $t_{biol}$:

- For radioactive compounds, we have to calculate the $t_{biol}$ because the mass of the isotope is usually on the nanogram scale and, when distributed throughout the body, and especially in the target organ, concentrations are in the picogram/ml range, much too small to measure directly.

- For non-radioactive compounds, we can measure the $t_{biol}$ directly. For example, assuming that a person is not allergic to penicillin, we could give 1,000 mg of the drug and then measure the amount present in the blood pool and in the urine since we administered such a large amount of the drug.

- One can also determine graphically what the biological half-life is. In the diagram below, the drug has a biological half-life of 7 hr. Unlike $t_{phys}$ which is boundless, $t_{biol}$ for commercially available radiopharmaceuticals is typically in the range of sec (ventilation study) to days (phosphate based bone agents).
• \( t_{\text{biol}} \) is affected by many external factors. Perhaps the two most important are hepatic and renal function. If kidneys are not working well, we would expect to see a high background activity on our scans. Also important is level of hydration. A poorly hydrated patient, even with normal renal function, will have a high background activity since limited urine is being produced, making it difficult to eliminate isotope that has not localized in the target organ.

• Each individual organ in the body has its own \( t_{\text{biol}} \) and the whole body also has a \( t_{\text{biol}} \) representing the weighted average of the \( t_{\text{biol}} \) of all internal organs and the blood pool. It is therefore very important to have a frame of reference. For example, do you need to know the \( t_{\text{biol}} \) of the drug in the liver or in the whole body?

• All drugs have a \( t_{\text{biol}} \), not just radioactive ones. Drug package inserts often refer to the half-time of clearance of a drug from the blood pool or through the kidneys.

• Since the whole body has a \( t_{\text{biol}} \) representing the weighted average of the \( t_{\text{biol}} \) of all internal organs, it will almost never equal that of an internal organ.

• It is a fallacy that physical and biological half-lives can never equal each other. Consider Tc-99m MAA, for which both \( t_{\text{phys}} \) and \( t_{\text{biol}} \) are equal to 6 hours.

• Consider two extreme cases: for Tc-99m Sulfur Colloid, the \( t_{\text{biol}} \) is considered to be infinitely long; the \( t_{\text{biol}} \) of Xe-133 gas in the lungs is measured in sec.
**Half Lives: Effective**

**Effective Half-Life** is defined as the period of time required to reduce the radioactivity level of an internal organ or of the whole body to exactly one half its original value due to both elimination and decay. It is designated $t_{\text{eff}}$ or $t_e$. There are a few things to note about the $t_{\text{eff}}$:

- The $t_{\text{eff}}$ can be measured directly. For example, one can hold a detection device 1 m from the patient’s chest and count the patient multiple times until the reading decreases to half of the initial reading. The patient is permitted to use the rest room between readings as needed, so both elimination and decay are taking place. The half-life being measured in this case is the $t_{\text{eff}}$.

- The range of $t_{\text{eff}}$ typically varies from sec to hr.

- $t_{\text{eff}}$ is affected by the same external factors that affect $t_{\text{biol}}$ since $t_{\text{eff}}$ is dependent upon $t_{\text{biol}}$.

**Mathematical Relationship**

There are three **special cases** that help to clarify the concept of effective half-life:

**If** $t_{\text{phys}} >>> t_{\text{biol}}$ **then** $t_{\text{eff}} \sim t_{\text{biol}}$

e.g., for a Xe-133 for pulmonary ventilation study, where $t_{\text{phys}} = 5.3$ days and $t_{\text{biol}} = 15$ sec, the study is over within a few minutes. It did not matter whether the Xe half-life was 5 hr, 5 days, or 5 weeks. They are all so long compared to the biological half-life that the effective half-life equals the biological half-life.

**Calculation:**

$$\frac{1}{457920 \text{ sec}} + \frac{1}{15 \text{ sec}} = \frac{1}{t_{\text{eff}}}$$

so... $t_{\text{eff}} = 15$ sec

**If** $t_{\text{biol}} >>> t_{\text{phys}}$ **then** $t_{\text{eff}} \sim t_{\text{phys}}$

e.g., for a liver scan with Tc-SC, $t_{\text{phys}} = 6$ hr, $t_{\text{biol}} = \text{Infinitely long}$. Because Tc-SC never clears the liver, then the only half-life that matters is the physical half-life,
Calculation:

\[
\frac{1}{\infty} + \frac{1}{6 \text{ hr}} = \frac{1}{t_{\text{eff}}}
\]

and... \( t_{\text{eff}} = 6 \text{ hr} \)

There is a third special case.

If \( t_{\text{biol}} = t_{\text{phys}} \) then \( t_{\text{eff}} = 1/2 \ t_{\text{biol}} = 1/2 \ t_{\text{phys}} \)

Example: for Tc-MAA for pulmonary perfusion imaging, \( t_{\text{phys}} = 6 \text{ hr}, \ t_{\text{biol}} = 6 \text{ hr}, \) then \( t_{\text{eff}} = 3 \text{ hr} \).

Calculation:

\[
\frac{1}{6 \text{ hr}} + \frac{1}{6 \text{ hr}} = \frac{1}{3 \text{ hr}}
\]

and... \( t_{\text{eff}} = 3 \text{ hr} \)

In most other cases, one has to mathematically solve for the desired half-life since they are not special cases.

**Problem 1.**

I-131 sodium iodide has a \( t_{\text{biol}} \) of 24 d. What is \( t_{\text{eff}} \)?

\[
\frac{1}{t_{\text{eff}}} = \frac{1}{t_{\text{phys}}} + \frac{1}{t_{\text{biol}}} = \frac{1}{8} + \frac{1}{24} = \frac{1}{6} \text{ so... } t_{\text{eff}} = 6 \text{ d}
\]

**Problem 2.**

A Tc-99m compound has a \( t_{\text{eff}} = 1 \) hr. What is \( t_{\text{biol}} \)?

\[
\frac{1}{t_{\text{biol}}} = \frac{1}{t_{\text{eff}}} - \frac{1}{t_{\text{phys}}} = \frac{1}{1} - \frac{1}{6} = \frac{5}{6} \text{ so... } t_{\text{biol}} = 1.2
\]
Problem 3.

A radiopharmaceutical has a biological half-life of 4.00 hr and an effective half-life of 3.075 hr. What isotope was used?

\[ \frac{1}{t_{\text{phys}}} = \frac{1}{t_{\text{eff}}} - \frac{1}{t_{\text{biol}}} = \frac{1}{3.075} - \frac{1}{4.00} = 0.0752033 \]

Therefore \( t_{\text{phys}} \) = 13.3 hr and the radioisotope is I-123

Radiation Dosimetry Equation

The equation below is commonly used for radiation dosimetry calculations.

You’ll observe that the only half-life in the equation is the effective half-life.

\[ D_\gamma = 73.8 \cdot E_\gamma \cdot f_\gamma \cdot C_0 \cdot 1.443 \cdot t_{\text{eff}} \cdot \varphi \text{ Rads} \]